SIM-Resident Continuous-Field Execution as a Universal Operator–Boundary Compiler: Exact Operator Stacks, DPDE Shape Forcing, Robin/DtN Resonance Gates, and Lagrangian Solver Experiments

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Abstract

This paper consolidates a unified analytic framework developed across the project: a SIM-resident control core driving a continuous-field screen, augmented by a Fourier-plane operator stage and programmable Robin boundary admittance, yields a universal *operator–boundary compiler*. The platform replaces raster buffers and discretized state tables with compact, equation-defined program states that generate spatiotemporal fields directly. Within this formalism we derive four interlocking result families. (i) An exact operator library implemented as compositions of real-space multiplication masks and Fourier-space spectral masks, enabling closed-form solvers and propagators for broad classes of constant-coefficient PDEs and producing special-function structure (e.g. Bessel/Hankel modes) as native radial identities. (ii) An exact DPDE shape-forcing compiler: for any analytic target geometry encoded as a field, the sustaining source terms required to make that geometry an exact steady solution are obtained by differentiation identities; articulated geometries follow from analytic implicit constructions. (iii) A Robin/DtN resonance-gate calculus: programmable boundary admittance yields explicit scattering and mode-selection laws, moving resonance poles and controlling Q-factors without altering the bulk operator family. (iv) A unified Lagrangian closure: bulk operator stacks and Robin boundaries arise from a single action principle, enabling a "Lagrangian solver" view that couples world dynamics to an observer subsystem written in explicit qualia coordinates, and supporting "physics-in-a-box" experiments where modified dispersions, fractional operators, heterogeneous couplings, and horizon-like boundaries are implemented as analytically specified masks and boundary terms. The result is a coherent end-to-end compilation chain from analytic specifications to realized fields, with applications spanning computation, control, sensing, experimental physics, and creative operator-defined media.

1 Introduction

Conventional computing, simulation, and display pipelines typically rely on explicit high-dimensional buffers: pixel arrays for rendering, discretized grids for PDE evolution, and large tables for control and measurement. This project advances a different primitive: *continuous fields generated from compact analytic specifications*. In this approach, a SIM-resident payload stores and updates a low-dimensional program state—a parameter set defining masks, symbols, and boundary laws—and the physical substrate executes that state as spatiotemporal fields rather than as precomputed buffers.

The central consolidation developed here is to treat the continuous-field screen as an exact *real-space multiplication operator* acting on fields, while a lens-plane (Fourier-plane) stage implements exact *spectral multiplication*. These two capabilities form the canonical pseudodifferential skeleton

$$\text{(real mask)} \circ \text{(Fourier mask)} \circ \text{(real mask)} \circ \cdots,$$

which is sufficient to express a wide class of analytic operators used across signal processing, PDE theory, resonance engineering, and field dynamics. Simultaneously, programmable Robin boundary admittance provides a second axis of control: boundary conditions become adjustable operator constraints, naturally expressed through Dirichlet-to-Neumann (DtN) maps. Together, real-space masks, Fourier-space masks, and Robin/DtN boundary programming define a closed analytic triad.

Three structural consequences follow immediately.

(i) **Operator computation without rasterization.** Constant-coefficient PDE inverses, propagators, derivatives, and convolution kernels become explicit spectral multipliers; special-function structure emerges from transform identities rather than from lookup tables.

(ii) **Geometry as a compiled invariant.** DPDE constructions are recast as compilation: a target analytic geometry field determines, by identity, the sustaining sources and coefficients that make it an exact steady solution of a chosen PDE family, with an analytic stability interface via linearized operator pencils.

(iii) **Boundary programming as a resonance gate.** Robin admittance yields explicit scattering laws and cavity eigenconditions; DtN maps convert boundary control into resonance and mode-selection constraints, enabling tunable Q-factor and pole placement at the boundary level.

A further consolidation is achieved by variational closure: bulk operator stacks and boundary gates arise from a single action principle, allowing a "Lagrangian solver" paradigm that couples dynamics, observation, and parameter programming within stationary-action equations. This, in turn, supports a "physics-in-a-box" methodology in which modified dispersions, fractional and nonlocal operators, heterogeneous couplings, and horizon-like boundaries are programmed analytically as masks and boundary terms for controlled scientific experiments.

Organization. The remainder of the paper proceeds as follows. Section 2 formalizes the operator–boundary primitives and establishes the exact compilation laws for mask stacks and boundary gates. Section 3 derives the Helmholtz/resolvent and special-function operator stacks, including radial Bessel-mode synthesis. Section 4 develops the DPDE shape-forcing compiler, including an analytic articulated-geometry construction and a stability interface. Section 5 presents the Robin/DtN resonance-gate calculus with explicit scattering and eigencondition laws. Section 6 provides the unified Lagrangian closure and formulates Lagrangian-solver applications, including an observer subsystem in qualia coordinates and a physics-in-a-box experimental framework. A concluding section summarizes implications and outlines a forward research agenda.

2 Creative Frontiers: Novel Proposals and Cross-Domain Advanced Applications

This section collects forward-looking proposals enabled by the operator–boundary compiler. The emphasis is on *creative minds* and *novel ideas* that become technically meaningful once one accepts the central premise: a SIM-resident analytic program state $\Theta(t)$ generates (i) bulk operator stacks, (ii) boundary gates, (iii) DPDE sustaining fields, and (iv) observer–qualia couplings, all within a single Lagrangian closure. The proposals below are framed as research directions with precise mathematical handles, not marketing narratives.

2.1 Domain-agnostic principle: "Program the operators, not the pixels"

The unifying innovation is representational: store and manipulate a compact analytic state $\Theta(t)$ and treat the physical platform as an executor of operator stacks and boundary laws. In this paradigm, *rendering*, *computation*, and *physical control* are different readings of the same object:

$$\Theta(t) \quad \Longrightarrow \quad \big(\mathcal{P}(\Theta),\, \kappa(\Theta),\, S(\Theta),\, w^{\alpha}(\Theta)\big).$$

Once this shift is made, new application classes become natural because many problems across domains reduce to constructing the right operators, boundary conditions, and sources.

2.2 Mathematics as a physical medium: special-function synthesis and operator galleries

(i) Special-function galleries as spectral design. Section **??** already shows that Bessel families emerge as native radial transform identities. This can be generalized into a *special-function gallery*:

- **Bessel/Hankel modes:** annular spectral selection and angular phase factors yield J_m families and vortex modes.
- **Airy-type structures:** cubic-phase spectral masks generate Airy profiles as exact Fourier integrals.
- **Hermite–Gaussian families:** quadratic-phase propagation and polynomial-weighted masks generate Hermite modes in paraxial analogs.

The proposal is a curated "operator museum" where each exhibit is a closed-form mask specification; the platform becomes a demonstrator of analytic transform identities *as a physical phenomenon.*

(ii) Operator composition as a language for creative computation. Because stacks $M_a \mathcal{O}_H M_b \cdots$ form a compositional language, creators can design new transforms by combining "atoms" (local masks) with "lenses" (spectral masks). This suggests a new category of creative tools: *operator editors* where artists and scientists manipulate functional forms instead of waveforms or images.

2.3 New control metaphors: DPDE shape compilation for robotics, design, and architecture

(i) "Shape-to-force" compilation for soft robotics. Section **??** gives an explicit compiler: choose a geometry Φ and obtain sustaining fields S. This yields a new design workflow:

$$\text{desired shape} \;\Rightarrow\; Q_0 = \sigma(\Phi) \;\Rightarrow\; S = -\Delta Q_0 + \mu Q_0 \;\Rightarrow\; \text{actuation field.}$$

The novel proposal is to treat soft robotic actuation as a steady-state PDE enforcement problem rather than a trajectory-control problem. Fingers, tendons, and joints become analytic components in Φ, and "motion" becomes time dependence in $\Theta(t)$.

(ii) Computational fabrication by PDE enforcement. Architectural surfaces and material microstructures can be specified as invariant manifolds of elliptic operators (or semilinear variants). The sustaining field becomes a *fabrication recipe* for spatially varying stress, heating, curing, or deposition profiles. This proposes a bridge between analytic PDE design and programmable manufacturing.

(iii) Geometry morphing without remeshing. Because $\Phi(x; \Theta(t))$ is analytic, continuous morphing (e.g. hand closing, grasp formation) is implemented by updating $\Theta(t)$, not by remeshing a model. This is a direct computational advantage for animation, CAD, and digital twins when the output medium is an operator executor.

2.4 Resonance gates as programmable physics: sensing, communications, and security

(i) Mode-selective filters as boundary programs. Robin/DtN gates provide a boundary-level filter bank: by shaping $\kappa(x)$, boundary harmonics are selected or suppressed. This offers a new synthesis route for:

- **Ultra-compact filters** (frequency/mode selection via boundary programming),
- **Adaptive resonators** (dynamic $\kappa(x, t)$ as a tuning knob),
- **Programmable Q-factors** (pole movement via boundary admittance changes).

(ii) Physical-layer security by boundary-keyed operators. Because the realized transfer function depends on the operator stack and boundary admittance, one can treat $\Theta(t)$ as a cryptographic key controlling the physical transform. The same input can map to different outputs under different programmed $\Theta(t)$, yielding *hardware-level keyed transforms* (a physical-layer obfuscation/locking concept expressed purely in operator algebra).

(iii) "Horizon" boundaries as laboratory metaphors. By implementing absorbing or radiative Robin boundaries, one can build tabletop analogs of open-system dynamics: controlled leakage, effective irreversibility, and resonance decay become programmable. This is not a claim of cosmological identity; it is a precise operator-boundary experiment platform.

2.5 Observer–qualia systems: cognition, education, and human-in-the-loop science

(i) Qualia coordinates as structured internal representations. The observer model of Section **??** treats perception as dynamics in a low-dimensional state $q(t)$ driven by mismatch with measured features $y^\alpha(t)$. A creative application is *interactive cognitive instrumentation*: the system can be programmed so that different feature sets w^α correspond to different "ways of seeing" the same field.

(ii) Education: making abstract operators tangible. An operator–boundary compiler can serve as an educational instrument where students interact with:

$$\text{symbol } \omega(k) \quad \Longrightarrow \quad \text{observable dynamics.}$$

This turns Fourier symbols, boundary conditions, and Green operators into visually and experimentally accessible objects.

(iii) Human-in-the-loop discovery. Let the human adjust $\Theta(t)$ directly (or via qualia-driven controls), exploring a space of operator stacks that produce interesting invariants, stable shapes, or resonance regimes. This creates a new discovery loop where intuition acts on operators rather than on raw data.

2.6 "Physics-in-a-box" as a platform for scientific creativity

(i) Alternate dispersions and synthetic media. Programming $\omega(k)$ and interleaving M_a masks enables synthetic dispersions and heterogeneous couplings. Scientists can explore how wave packets, stability thresholds, and resonance structures change under modified symbols, with every modification traceable to an analytic specification.

(ii) Fractional and nonlocal physics as first-class citizens. Fractional Laplacians and nonlocal operators are naturally spectral. Hence the platform is unusually well-suited to nonlocal physics experiments: effects are introduced by specifying analytic symbols rather than by discretized stencils.

(iii) Boundary-driven phase transitions and stability cliffs. Because the boundary is programmable, one can explore regimes where small changes in κ cause qualitative changes: mode switching, resonance splitting, or stability loss. This offers a controlled environment for studying bifurcation-like phenomena in open systems.

2.7 Creative media and art: operator cinema and equation-defined narratives

(i) Operator cinema. Instead of a frame buffer, one stores a story as a time sequence of operator programs $\Theta(t)$: a narrative defined by evolving symbols, boundary gates, and DPDE shapes. The resulting visuals are not pre-rendered; they are executed live as analytic fields.

(ii) Equation-defined choreography. DPDE shape fields can represent dancers, props, or stage boundaries, with motion defined by parameter flows $\Theta(t)$. Resonance gates can produce synchronized oscillatory motifs. The proposal is an art medium where choreography is a flow on parameter space rather than a list of keyframes.

(iii) "Qualia installations." Couple an observer model to the same fields it measures and render its internal state $q(t)$ as a visible or audible field. This creates installations where the system's "interpretation" is part of the artwork, formally defined by (**??**) and measurement functionals (**??**).

2.8 A research agenda for creative minds

The proposals above suggest a coherent research agenda:

1. **Operator design languages:** formal grammars for stacks (4) and their equivalence classes (normal forms, identifiability, and minimal parameterizations).
2. **Stability-by-design:** integrate the DPDE stability pencil (**??**) into the compilation loop, so that generated shapes and dynamics carry analytic stability certificates.
3. **Boundary gate libraries:** catalog κ patterns as reusable mode selectors, absorbers, and Q-tuners with closed-form scattering properties.
4. **Observer toolkits:** develop families of qualia maps $h(q)$ and feature masks w^α that correspond to interpretable internal coordinates for different domains (vision, acoustics, mechanics, fluids).

5. **Physics-in-a-box protocols:** define experiment templates where "physics variants" are specified by a small set of symbol/boundary changes with measurable consequences.

2.9 Summary

The operator–boundary compiler is not limited to a single domain. By providing a closed analytic interface for bulk operators, boundary gates, DPDE forcing, and observer dynamics, it creates a versatile sandbox for creative research: mathematics made physical, control reimagined as operator programming, resonance used as a gate and a sensor, observation framed as qualia dynamics, and modified physics explored as a programmable experiment.

2.10 Fourier convention and fundamental identities

Let $f : \mathbb{R}^d \to \mathbb{C}$ be integrable (or a tempered distribution where appropriate). We fix the Fourier transform

$$\hat{f}(k) \;=\; (\mathcal{F}f)(k) \;:=\; \int_{\mathbb{R}^d} f(x)\, e^{-2\pi \mathrm{i}\, x\cdot k}\, \mathrm{d}x, \qquad f(x) \;=\; (\mathcal{F}^{-1}\hat{f})(x) \;:=\; \int_{\mathbb{R}^d} \hat{f}(k)\, e^{2\pi \mathrm{i}\, x\cdot k}\, \mathrm{d}k. \tag{1}$$

With (1), the differentiation rules are exact:

$$\mathcal{F}(\partial_{x_j} f)(k) \;=\; (2\pi \mathrm{i} k_j)\, \hat{f}(k), \qquad \mathcal{F}(\Delta f)(k) \;=\; -(2\pi)^2 |k|^2\, \hat{f}(k). \tag{2}$$

2.11 Real-space and Fourier-space mask operators

Definition 1 (Multiplication mask)**.** For a measurable function $a : \mathbb{R}^d \to \mathbb{C}$, define the multiplication (mask) operator

$$(M_a f)(x) := a(x)\, f(x).$$

Definition 2 (Fourier-plane mask (lens operator))**.** For a measurable function $H : \mathbb{R}^d \to \mathbb{C}$, define the Fourier-plane mask operator

$$\mathcal{O}_H \;:=\; \mathcal{F}^{-1}\, M_H\, \mathcal{F}. \tag{3}$$

Remark 1. The operator $\mathcal{O}_H$ is convolution by $h = \mathcal{F}^{-1}H$ in the distributional sense: $\mathcal{O}_H f = h * f$. Thus M_a provides programmable *local* modulation, while $\mathcal{O}_H$ provides programmable *nonlocal* transforms.

2.12 The universal operator stack

The platform's computational core is the ability to compose real and Fourier masks into finite products. We therefore formalize:

Definition 3 (Stack operator algebra)**.** An operator stack is any finite composition of the form

$$\mathcal{T} \;=\; M_{a_0}\, \mathcal{O}_{H_1}\, M_{a_1}\, \mathcal{O}_{H_2} \cdots \mathcal{O}_{H_n}\, M_{a_n}, \tag{4}$$

with coefficient masks $a_j(x)$ and spectral masks $H_j(k)$.

Proposition 1 (Exact realization of constant-coefficient PDE inverses)**.** *Let $P(D)$ be a constant-coefficient linear differential operator on $\mathbb{R}^d$ with Fourier symbol $P(2\pi \mathrm{i}k)$, meaning $\mathcal{F}(P(D)u) = P(2\pi \mathrm{i}k)\hat{u}$. If $P(2\pi \mathrm{i}k)$ is nonzero on the spectral support under consideration, then the solution of*

$$P(D)u = f$$

is given exactly by a single Fourier-plane mask:

$$u \;=\; \mathcal{F}^{-1}\!\left(\frac{1}{P(2\pi \mathrm{i}k)}\,\hat{f}\right) \;=\; \mathcal{O}_H\, f, \qquad H(k) = \frac{1}{P(2\pi \mathrm{i}k)}. \tag{5}$$

Corollary 1 (Helmholtz resolvent mask)**.** *For $(-\Delta + \mu)u = f$ on $\mathbb{R}^d$ with parameter $\mu \in \mathbb{C}$, the resolvent is the spectral mask*

$$u = \mathcal{O}_{H_\mu} f, \qquad H_\mu(k) = \frac{1}{(2\pi)^2|k|^2 + \mu}. \tag{6}$$

2.13 Radial geometry and Bessel/Hankel kernels as native masks

In $d = 2$, for radial data the inverse Fourier transform reduces to a Hankel-type (Bessel) integral. In particular, the kernel J_0 appears canonically in the reconstruction of radial functions. This yields a direct special-function generator by spectral design: a ring-supported spectrum produces a Bessel-profile field.

Proposition 2 (Ring spectrum generates a Bessel J_0 field (distributional identity))**.** *In $d = 2$, let $\hat{f}(k)$ be radial and concentrated on the circle $|k| = \rho_0$ in the distributional sense. Then the corresponding real-space field is proportional to $J_0(2\pi\rho_0 r)$ with $r = |x|$. Consequently, a radial Fourier-plane mask designed to select annular spectral content yields Bessel-structured outputs as an exact transform identity.*

2.14 Programmable Robin admittance as a boundary compiler

The boundary-control axis is expressed through Robin conditions. Let $\Omega \subset \mathbb{R}^d$ have boundary $\Gamma = \partial\Omega$ and outward normal derivative ∂_n.

Definition 4 (Robin boundary law)**.** A Robin boundary condition is

$$\partial_n u + \kappa(x,t)\, u = 0 \quad \text{on } \Gamma, \tag{7}$$

where κ is the boundary admittance (possibly complex, and possibly space–time dependent).

Robin boundary programming yields explicit scattering and resonance control. In the simplest 1D half-line model, Robin admittance produces a closed-form reflection coefficient and thus an analytic gate law.

Proposition 3 (Exact reflection coefficient under Robin admittance)**.** *On the half-line $x \geq 0$, consider a monochromatic field $u(x) = e^{\mathrm{i}kx} + Re^{-\mathrm{i}kx}$ with boundary condition $u'(0) + \kappa u(0) = 0$. Then*

$$R(k;\kappa) = \frac{\mathrm{i}k - \kappa}{\mathrm{i}k + \kappa}. \tag{8}$$

Proposition 4 (Robin–Robin cavity eigencondition)**.** *On $[0, L]$, consider $u'' + \alpha^2 u = 0$ with boundary laws $u'(0) + \kappa_0 u(0) = 0$ and $u'(L) + \kappa_L u(L) = 0$. Nontrivial solutions exist if and only if*

$$\tan(\alpha L) = \frac{\alpha(\kappa_L - \kappa_0)}{\alpha^2 + \kappa_0\kappa_L}. \tag{9}$$

2.15 From primitives to the four application families

Section **??** establishes the closed analytic substrate used throughout the remainder of the manuscript:

1. Helmholtz and related resolvents are single spectral masks (6), and radial geometry yields Bessel/Hankel structures (Proposition 2).
2. DPDE shape forcing is compiled by selecting a target analytic field Q_0 and defining its sustaining source by an identity of the form $S = -\Delta Q_0 + \mu Q_0$ (derived later), which then maps into screen control fields via an invertible transmission–control law.
3. Robin/DtN resonance gates are boundary masks with explicit reflection and spectrum laws (8)–(9).
4. A unified Lagrangian principle generates both bulk operator dynamics and Robin boundaries, and is then extended with an observer module carrying qualia coordinates and with "physics-in-a-box" modified dispersions specified directly by spectral symbols.

3 Classical "Teleportation": Exact State Transfer as an Operator–Boundary Program

In this section we derive a classical-scale analog of "teleportation" in the only sense that is mathematically defensible at classical scale: *exact transfer of a field state* (amplitude/phase information, or a mode state) from a source region $\mathcal{A}$ to a target region $\mathcal{B}$, without requiring a pixel buffer or numerical simulation. The transfer is expressed as an *operator identity* implemented by (i) bulk operators (Fourier masks / propagators), (ii) boundary gates (Robin/DtN programming), and (iii) optional observer-driven feedback (phase conjugation / time reversal). No approximations are used.

3.1 What "teleportation" means here (and what it does not)

We do *not* claim transport of matter or discontinuous spacetime relocation. The classical objective is:

$$\boxed{\text{Given a classical field state on } \mathcal{A}, \text{ produce the same state on } \mathcal{B} \text{ (up to a known, invertible gauge).}}$$

This is a *state-transfer* or *field-relocation* problem, and it admits exact analytic solutions under linear wave/field dynamics and programmable boundary conditions.

3.2 Operator primitives for spacetime translation

Let $f : \mathbb{R}^d \to \mathbb{C}$ be a field snapshot. Define the spatial translation operator by

$$(\mathcal{T}_{\Delta x} f)(x) := f(x - \Delta x). \tag{10}$$

This operator has two exact realizations:

(i) Differential-operator form.

$$\boxed{\mathcal{T}_{\Delta x} = \exp\big(\Delta x \cdot \boldsymbol{\nabla}\big),} \tag{11}$$

since $\exp(\Delta x \cdot \boldsymbol{\nabla})$ is the Taylor-shift operator.

(ii) Fourier-mask form (lens-plane realization). Using the Fourier convention (1),

$$\boxed{\mathcal{T}_{\Delta x} = \mathcal{F}^{-1} M_{H_{\Delta x}} \mathcal{F}, \qquad H_{\Delta x}(k) = e^{-2\pi \mathrm{i}\, k\cdot \Delta x}.} \tag{12}$$

Thus spatial teleportation of a *field snapshot* is literally a single spectral phase mask.

Similarly, for a time signal $g(t)$, define time translation

$$(\mathcal{S}_{\Delta t} g)(t) = g(t - \Delta t) = e^{\Delta t\, \partial_t} g(t).$$

In the temporal Fourier domain (frequency ω),

$$\boxed{\mathcal{S}_{\Delta t} = \mathcal{F}_t^{-1} M_{e^{-\mathrm{i}\omega\Delta t}} \mathcal{F}_t.} \tag{13}$$

3.3 Teleportation through a known propagation channel: invertible operator transfer

Let a linear physical channel map a boundary/source excitation s on $\Gamma_{\mathcal{A}}$ to an observed field y on $\Gamma_{\mathcal{B}}$:

$$y = \mathcal{K}\, s, \tag{14}$$

where $\mathcal{K}$ is the channel operator (a Green operator, transfer operator, or scattering map). If $\mathcal{K}$ is invertible on the relevant subspace, then exact transfer is immediate:

$$\boxed{s = \mathcal{K}^{-1} y, \qquad \text{and to reproduce a desired } y_\star, \text{ choose } s_\star = \mathcal{K}^{-1} y_\star.} \tag{15}$$

This is classical "teleportation" as exact precompensation.

How the project implements $\mathcal{K}^{-1}$ analytically. When $\mathcal{K}$ arises from a constant-coefficient bulk operator (e.g. Helmholtz/wave propagation), its inverse is a spectral multiplier (Section **??**). When $\mathcal{K}$ is boundary-mediated, $\mathcal{K}^{-1}$ is expressed via DtN gates (Section **??**).

3.4 Boundary-operator statement: DtN teleportation via Robin gating

Fix a bulk operator $\mathcal{A}(\omega)$ at a frequency parameter ω and let $\Lambda(\omega)$ be the Dirichlet-to-Neumann map (Section **??**). On a boundary Γ, the Robin law is

$$(\Lambda(\omega) + \kappa)\phi = 0.$$

Now consider two boundary ports $\Gamma_{\mathcal{A}}$ and $\Gamma_{\mathcal{B}}$ coupled through the bulk. In a boundary basis, the coupled boundary data $(\phi_{\mathcal{A}}, \phi_{\mathcal{B}})$ obey a linear relation

$$\begin{pmatrix} \psi_{\mathcal{A}} \\ \psi_{\mathcal{B}} \end{pmatrix} = \begin{pmatrix} \mathcal{S}_{AA} & \mathcal{S}_{AB} \\ \mathcal{S}_{BA} & \mathcal{S}_{BB} \end{pmatrix} \begin{pmatrix} \phi_{\mathcal{A}} \\ \phi_{\mathcal{B}} \end{pmatrix}, \tag{16}$$

where ψ denotes the conjugate boundary flux (Neumann data) and $\mathcal{S}_{\bullet\bullet}$ are exact operator blocks determined by the bulk Green structure.

Imposing programmable Robin laws

$$\psi_{\mathcal{A}} + \kappa_{\mathcal{A}}\phi_{\mathcal{A}} = j_{\mathcal{A}}, \qquad \psi_{\mathcal{B}} + \kappa_{\mathcal{B}}\phi_{\mathcal{B}} = j_{\mathcal{B}},$$

yields a closed linear system for $(\phi_{\mathcal{A}}, \phi_{\mathcal{B}})$. Teleportation-as-transfer means: choose $\kappa_{\mathcal{A}}, \kappa_{\mathcal{B}}$ (and sources $j_{\mathcal{A}}$) so that

$$\boxed{\phi_{\mathcal{B}} = \mathcal{G}\, \phi_{\mathcal{A}}} \tag{17}$$

for a desired *transfer operator* $\mathcal{G}$ (ideally $\mathcal{G} = I$ up to a phase). Since all maps are linear operators, (17) reduces to an exact operator equation solvable by algebra on the boundary blocks.

3.5 A clean closed-form classical teleportation engine: two coupled resonant modes

The most transparent analytic "teleportation" mechanism is exact state swap between two classical resonant modes. Let $a(t), b(t) \in \mathbb{C}$ be complex amplitudes of two classical oscillators (or two selected field modes) with Hamiltonian

$$H = \omega\big(|a|^2 + |b|^2\big) + g\,(ab^* + a^*b), \qquad \omega \in \mathbb{R},\ g \in \mathbb{R}. \tag{18}$$

Hamilton's equations in complex form yield

$$\mathrm{i}\dot{a} = \omega a + gb, \qquad \mathrm{i}\dot{b} = \omega b + ga. \tag{19}$$

Define the rotating frame $\tilde{a}(t) = e^{\mathrm{i}\omega t}a(t)$, $\tilde{b}(t) = e^{\mathrm{i}\omega t}b(t)$, which removes the common ω term:

$$\mathrm{i}\dot{\tilde{a}} = g\tilde{b}, \qquad \mathrm{i}\dot{\tilde{b}} = g\tilde{a}. \tag{20}$$

Differentiate once more to get $\ddot{\tilde{a}} + g^2\tilde{a} = 0$ and similarly for $\tilde{b}$, hence the solution is an exact rotation:

$$\boxed{\begin{pmatrix}\tilde{a}(t)\\ \tilde{b}(t)\end{pmatrix} = \begin{pmatrix}\cos(gt) & -\mathrm{i}\sin(gt)\\ -\mathrm{i}\sin(gt) & \cos(gt)\end{pmatrix}\begin{pmatrix}\tilde{a}(0)\\ \tilde{b}(0)\end{pmatrix}.} \tag{21}$$

At time

$$t_\star = \frac{\pi}{2g}, \tag{22}$$

the matrix becomes (up to a phase) a swap:

$$\boxed{\tilde{a}(t_\star) = -\mathrm{i}\,\tilde{b}(0), \qquad \tilde{b}(t_\star) = -\mathrm{i}\,\tilde{a}(0).} \tag{23}$$

Thus, the state initially stored in mode a appears in mode b exactly (up to the known phase $-\mathrm{i}$), with no approximation. This is classical teleportation-as-swap.

Embedding into the operator–boundary compiler. Mode a can be chosen as a boundary/port mode on $\mathcal{A}$ and mode b as a boundary/port mode on $\mathcal{B}$, selected via Robin/DtN gates (Section **??**). The coupling g is realized by programming the inter-port transfer (an operator block like $\mathcal{S}_{BA}$), and the timing $t_\star$ is exact.

3.6 Field-level teleportation: mode expansion + exact swap per mode

Let a field on Ω decompose into a complete orthonormal mode basis $\{\varphi_j\}$ (bulk or boundary-selected):

$$\phi(x,t) = \sum_j q_j(t)\varphi_j(x).$$

Select a finite (or structured) subset of modes and pair each source-mode amplitude $q_j^{(\mathcal{A})}$ with a target-mode amplitude $q_j^{(\mathcal{B})}$. Applying the exact two-mode swap dynamics (19)–(23) independently per j yields

$$q_j^{(\mathcal{B})}(t_\star) = e^{\mathrm{i}\theta_j} q_j^{(\mathcal{A})}(0)$$

with known phases θ_j. The field on $\mathcal{B}$ is then the reconstructed mode sum. Because each step is an exact analytic mapping, the full transfer is exact on the selected subspace.

3.7 Observer-assisted teleportation: phase conjugation and time reversal as exact inverses

If the channel operator $\mathcal{K}$ is hard to invert directly, the project's observer machinery provides an exact alternative: *time reversal* or *phase conjugation*.

For wave equations generated from a time-reversal symmetric Lagrangian (Section **??**), if one records boundary data on Γ and re-injects the time-reversed conjugate data, the field refocuses to the source distribution. In operator terms, this is implementing an inverse (or adjoint inverse) of the channel on the relevant subspace:

$$\text{re-injection operator} \sim \mathcal{K}^\dagger \quad \Rightarrow \quad \mathcal{K}\mathcal{K}^\dagger \text{ acts as a refocusing projector.}$$

This is still purely analytic at the level of operators: it is an exact statement about adjoints, boundary pairings, and the propagator symmetry.

3.8 Summary: the classical teleportation stack

Classical "teleportation" is fully captured by exact operator equalities:

1. **Instant snapshot translation:** a single Fourier-phase mask implements $\mathcal{T}_{\Delta x}$ exactly (12).
2. **Channel precompensation:** if $y = \mathcal{K}s$ is invertible, choose $s_\star = \mathcal{K}^{-1} y_\star$ (15).
3. **Boundary-gated transfer:** use DtN/Robin programming to force a desired boundary transfer operator $\mathcal{G}$ (17).
4. **Exact resonant swap:** coupled-mode dynamics swaps states exactly at $t_\star = \pi/(2g)$ (23).
5. **Observer assistance:** time reversal / phase conjugation implements an adjoint-inverse refocusing operator on suitable subspaces.

This completes a purely analytic, no-approximation derivation of "teleportation" at classical scale as an operator–boundary program: the thing that moves is the *state* (field information), and the machinery that moves it is an exact composition of spectral masks, boundary gates, and (optionally) observer-driven inverse operators.